Recursion Subspace-Based Method for Bearing Estimation*

Lei HUANG†, Xia LI†, Liangwei CAI†, Shunjun WU††

† College of Information Engineering, Shenzhen University, Shenzhen, Guangdong 518060, China

†† National Lab of Radar Signal Processing, Xidian University, Xi'an 710071, China

E-mail: †lhuang@szu.edu.cn

Abstract In this paper, a recursion subspace-based method (RSM) is presented to estimate the bearings of incident signals impinging on a uniform linear array (ULA). Unlike the existing linear operation based methods, such as the signal subspace method without eigendecomposition (SUMWE) of Xin *et al*, that calculate the noise subspace, the RSM method finds a cleaner *signal subspace* by means of a successive recursion procedure. Thus, the RSM method has the advantages of computational simplicity and accurate estimation, in particular for a large array. Numerical results are presented to compare the performance the RSM method with that of the SUMWE methods.

key words: *Sensor array signal processing, Bearing estimation, Forward/backward spatial smoothing (FBSS).*

1. Introduction

High-resolution bearing estimation of incident signals impinging on an array of sensors is an important problem in many sensor systems, such as wireless communications, radar, sonar and so on. For high-resolution bearing estimations of narrowband signals, there have been a number of approaches including the maximum likelihood (ML) [1] and subspace-based methods. The ML methods yield the optimal bearing estimations by solving a multidimensional nonlinear minimization problem [1]. Unfortunately, the multidimensional nonlinear minimization procedure is rather computationally cumbersome, consequently implying that the ML methods are unsuitable for real-time implementations. In contrast to the ML methods, the subspace-based methods only yield the suboptimal bearing estimations but have the advantage of computational simplicity, and thereby have received considerable attention. As a result, the subspace-based methods, such as the MUSIC [2], have been widely investigated. However, the subspace-based methods in general rely on the estimate of a covariance matrix and its eigenvalue decomposition (EVD). Unfortunately, the procedure of estimating the covariance matrix and computing the eigenvalues is still computationally intensive and time-consuming, especially for a very large array, which implies that the subspace-based methods depending on eigendecomposition are unsuitable for some practical situations where the number of sensors is large and/or the directions of incident signals need to be tracked in an on line manner. Thus, the ability to accurately resolve the incident signals with low computational complexity becomes very crucial in the practical environments.

To reduce the computational burden of the classical subspace-based methods, many computationally efficient subspace-based methods for bearing estimation are developed, which do not rely on the EVD computation. To avoid the EVD of the covariance matrix, the linear operation based methods [3]-[6] partition the array response matrix in the absence of noise (or the covariance matrix in the presence of noise) into a signal or noise subspace. The bearing estimations are yielded in a manner similar to the MUSIC algorithm. Nevertheless, the linear operation based methods, such as the methods of bearing estimation without eigendecomposition (BEWE) [3], the projector method (PM) [4] and the subspace method without engendecomposition (SWEDE) [5], essentially involve the estimated covariance matrix and several times of complex matrix-matrix products, and thereby are still computationally intensive. While the subspace-based method without eigendecomposition (SUMWE) [6] requires relatively lower computational burden than the other linear operation based methods due to avoiding the estimated covariance matrix and is suitable for coherent signals, it is less accurate than the MUSIC estimator for uncorrelated signals because its working array aperture becomes small.

In this paper, we develop a recursion subspace-based method (RSM) for bearing estimation that has the advantage of computational simplicity and maintains the estimation accuracy of the bearing estimation methods. Unlike the existing subspace-based methods, the RSM method attains the *signal subspace* by means of a successive refinement procedure, and does not involve the estimated covariance matrix or its EVD. Therefore, the RSM method has the advantage of computational simplicity over the EVD-based methods.

2. Data Model and Basic Assumptions

Consider a uniform linear array (ULA) composed of M isotropic sensors. Impinging upon the array are p narrowband signals $\{u_1(t), u_2(t), \cdots, u_p(t)\}$ from distinct directions $\{\theta_1, \theta_2, \cdots, \theta_p\}$. The p narrow-band signal sources, centered around a known frequency w_0, are placed in the far field, and thereby the wavefronts can be approximated as planar. For simplicity, we also assume that the sources and the sensors are in the same plane. Thus, employing complex envelope representation, the $M \times 1$ received vector of the array can be expressed as

$$\boldsymbol{y}(t)=[y_1(t), \cdots, y_M(t)]^T=\sum_{i=1}^{p} \boldsymbol{a}(\theta_i)u_i(t) + \boldsymbol{n}(t), \quad (1)$$

where $y_m(t)=\sum_{i=1}^{p} u_i(t)e^{j\omega_0\tau_{m-1}(\theta_i)} + n_m(t)$ is the received noisy signal at the mth sensor, named as the mth sensor data of the array, $\boldsymbol{a}(\theta_i) = \left[1, e^{j\omega_0\tau_1(\theta_i)}, \cdots, e^{j\omega_0\tau_{M-1}(\theta_i)}\right]^T$ is the

*This work was supported by the Guangdong Natural Science Foundation (No. 7301317) and the SZU R/D Foundation (No. 200748).

steering vector of the array toward direction θ_i, $u_i(t)$ is the scalar complex waveform referred to as the ith signal, $\tau_{m-1}(\theta_i) = \frac{d}{c}(m-1)\sin\theta_i$ denotes the propagation delay between the first sensor (the reference point) and the mth sensor to a wavefront impinging from direction θ_i, where c denotes the propagation speed and d is the distance between two adjacent sensors, $n_m(t)$ is the additive noise at the mth sensor, and $(\cdot)^T$ is transpose operation.

In matrix notation, Equation (1) can be rewritten more compactly as

$$\boldsymbol{y}(t) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{u}(t) + \boldsymbol{n}(t), \tag{2}$$

where $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_p)]$ is the array response matrix.

Throughout the paper, we make the following basic assumptions on the sensor data model:

A1: The array response matrix $\boldsymbol{A}(\boldsymbol{\theta})$ is unambiguous. That is to say, the array response vectors $\{\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_p)\}$ are linearly independent for any set of distinct incident angles $\{\theta_1, \theta_2, \cdots, \theta_p\}$, which indicates that the matrix $\boldsymbol{A}(\boldsymbol{\theta})$ is of full rank.

A2: We assume all the signals are zero-mean, jointly stationary, temporally complex white Gaussian random processes. The covariance matrix of the signal $\boldsymbol{u}(t)$ is given as

$$\boldsymbol{R}_u = E\big[\boldsymbol{u}(t)\boldsymbol{u}^H(t)\big] = \lim_{N\to\infty} \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{u}(t)\boldsymbol{u}^H(t),$$

where $(\cdot)^H$ is Hermitian transpose and $t = 1,\cdots,N$ denote the sampled instants. The number of sources p is assumed to be accurately estimated by the method proposed in [7] , and satisfies the inequality $p < M/2$. In addition, the signals are uncorrelated with the additive noise.

A3: The background noise is a temporally and spatially white Gaussian random process with zero mean and the following second moments:

$$E\left[\boldsymbol{n}(t)\boldsymbol{n}^H(s)\right] = \sigma_n^2 \delta_{t,s}\boldsymbol{I}_M$$
$$E\left[\boldsymbol{n}(t)\boldsymbol{n}^T(s)\right] = \boldsymbol{0},$$

where σ_n^2 is noise variance, $\boldsymbol{I}_M$ represents the $M \times M$ identity matrix, $\delta_{t,s}$ denotes Kroecker delta which is 1 for $t = s$ and 0 for $t \neq s$.

3. RSM for Bearing Estimation

In this section, we assume that the received sensor data $\boldsymbol{y}(t)$ sampled at N time instants $t = 1, 2, \cdots, N$ are available. Meanwhile, the signals $\boldsymbol{u}(t)$ are assumed to be incoherent. In the next section, the RSM will be extended to the coherent signal case by means of using the FBSS method to decorrelate the coherency of the signals.

3.1 Bearing Estimation

To begin with, using the shift-invariance property of the ULA, we define a new observation data from (1) and (3) as $\boldsymbol{y}_0(t) = [y_2(t), \cdots, y_M(t)]^T = \boldsymbol{A}_1(\boldsymbol{\theta})\boldsymbol{D}\boldsymbol{u}(t) + \tilde{\boldsymbol{n}}(t)$, where $\boldsymbol{A}_1(\boldsymbol{\theta})$ consists of the first $M-1$ rows of $\boldsymbol{A}(\boldsymbol{\theta})$, $\boldsymbol{D} = \text{diag}\left(e^{jw_0 d/c\sin\theta_1}, \cdots, e^{jw_0 d/c\sin\theta_p}\right)$ and $\tilde{\boldsymbol{n}}(t) = [n_2(t), n_3(t), \cdots, n_M(t)]^T$. To calculate the signal subspace by a successive refinement procedure, we must also define a reference signal by $d_0(t) = y_1(t) = \boldsymbol{u}^T(t)\boldsymbol{1} + n_1(t)$, where $\boldsymbol{1} = [1, 1, \cdots, 1]^T$. In the sequel, we can calculate the cross-correlation between the new observation data and the reference signal, namely the initial information for the refinement procedure:

$$\begin{aligned}\boldsymbol{r}_{y_0 d_0} &= E\left[\boldsymbol{y}_0(t)d_0^*(t)\right] \\ &= \left[r_{2,1}, r_{3,1}, \cdots, r_{M,1}\right]^T \\ &= \boldsymbol{A}_1(\boldsymbol{\theta})\boldsymbol{D}\boldsymbol{R}_u\boldsymbol{1} \\ &\triangleq \boldsymbol{A}_1(\boldsymbol{\theta})\boldsymbol{\beta},\end{aligned} \tag{3}$$

where $r_{i,1} = E\left[y_i(t)y_1^*(t)\right] (i = 2, 3, \cdots, M)$, and $\boldsymbol{\beta} = \boldsymbol{D}\boldsymbol{R}_u\boldsymbol{1}$. It is easy to prove that $\boldsymbol{\beta} \neq \boldsymbol{0}$ since both $\boldsymbol{D}$ and $\boldsymbol{R}_u$ are the nonsingular metrics. Therefore, the cross-correlation $\boldsymbol{r}_{y_0 d_0}$ is a linear combination of all the direction vectors $\boldsymbol{a}_1(\theta_i)(i = 1, 2, \cdots, p)$ with nonzero coefficients. Meanwhile, it is indicated in (4) that the additive noise has been efficiently eliminated while calculating $\boldsymbol{r}_{y_0 d_0}$. Since the cross-correlation $\boldsymbol{r}_{y_0 d_0}$ is able to capture the signal information, we use it to define the following matched filter

$$\boldsymbol{h}_1 = \frac{\boldsymbol{r}_{y_0 d_0}}{\|\, \boldsymbol{r}_{y_0 d_0} \|_2}. \tag{4}$$

Partitioning the new sensor data $\boldsymbol{y}_0(t)$ with the matched filter $\boldsymbol{h}_1$ in a manner similar to that of the multistage Wiener filter (MSWF) [8], we attain the desired signal $d_i(t)$ and its orthogonal component $\boldsymbol{y}_i(t)$ at the ith stage by

$$d_i(t) = \boldsymbol{h}_i^H \boldsymbol{y}_{i-1}(t) \tag{5}$$

and

$$\begin{aligned}\boldsymbol{y}_i(t) &= \boldsymbol{y}_{i-1}(t) - \boldsymbol{h}_i d_i(t) \\ &= \boldsymbol{y}_{i-1}(t) - \boldsymbol{h}_i\boldsymbol{h}_i^H\boldsymbol{y}_{i-1}(t) \\ &= \boldsymbol{B}_i\boldsymbol{y}_{i-1}(t),\end{aligned} \tag{6}$$

where $\boldsymbol{B}_i = \boldsymbol{I} - \boldsymbol{h}_i\boldsymbol{h}_i^H$ is the blocking matrix, and $\boldsymbol{h}_i$ is the matched filter updated as

$$\boldsymbol{h}_i = \frac{E[\boldsymbol{y}_{i-1}(t)d_{i-1}^*(t)]}{\|E[\boldsymbol{y}_{i-1}(t)d_{i-1}^*(t)]\|_2}. \tag{7}$$

It is indicated in (6)-(8) that the desired signal $d_i(t)$ is obtained by pre-filtering the observation data $\boldsymbol{y}_{i-1}(t)$ with the matched filters $\boldsymbol{h}_i$, but annihilated by the blocking matrix $\boldsymbol{B}_i$. The observation data is partitioned stage-by-stage in the same refinement manner. As a result, we obtain the pre-filtering matrix $\boldsymbol{T}_s = [\boldsymbol{h}_1, \ldots, \boldsymbol{h}_p]$ by p successive recursions.

Lemma 1: The p matched filters $\boldsymbol{h}_i$ $(i = 1, 2, \cdots, p)$ span the same range subspace of $\boldsymbol{A}_1(\boldsymbol{\theta})$, namely

$$\boldsymbol{T}_s = \boldsymbol{A}_1(\boldsymbol{\theta})\boldsymbol{H}, \tag{8}$$

where $\boldsymbol{H} \in \mathbb{C}^{p\times p}$ is a full-rank matrix and $\boldsymbol{T}_s = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_p]$.

The proof of Lemma 1 can be seen in [7] and [9].

It is easy to observe from (9)-(11) that the matched filters $\boldsymbol{h}_i (i = 2, 3, \cdots, p)$ are refined from the observation data $\boldsymbol{y}_1(t) = \boldsymbol{y}_0(t) - \boldsymbol{h}_1 \boldsymbol{h}_1^H \boldsymbol{y}_0(t)$ and the desired signal $d_1(t) = \boldsymbol{h}_1^H \boldsymbol{y}_0(t)$. Consequently, the calculation accuracy of the matched filters $\boldsymbol{h}_i (i = 2, 3, \cdots, p)$ essentially relies on the initial information for the refinement procedure, $\boldsymbol{r}_{y_0 d_0}$. Note that $\boldsymbol{r}_{y_0 d_0}$ (See (4)) is able to capture the signal information while efficiently suppressing the additive noise. Therefore, the estimated signal subspace becomes cleaner than that calculated by the conventional EVD-based methods because the EVD-based methods cannot eliminate the additive noise while calculating the eigenvectors.

Lemma 1 indicates that the signal subspace can be generated by means of the recursion procedure. Therefore, after obtaining the signal subspace, it is straightforward to employ it in the signal subspace based methods for bearing estimation, such as the MUSIC and ESPRIT estimators. In this paper, we obtain the bearing estimates in a manner similar to that of the classical MUSIC method by searching p highest peaks of the spatial spectral function:

$$S(\theta) = \frac{1}{\boldsymbol{a}_1(\theta) \boldsymbol{P}_n \boldsymbol{a}_1(\theta)}, \tag{9}$$

where $\boldsymbol{P}_n = \boldsymbol{I}_{M-1} - \boldsymbol{T}_s \left(\boldsymbol{T}_s^H \boldsymbol{T}_s \right)^{-1} \boldsymbol{T}_s^H$.

Note that the dimension of the observation space is reduced from M to $M - 1$. This implies that the RSM method might not be as accurate as the classical MUSIC estimator in direction-of-arrival (DOA) estimation. Nevertheless, for a reasonably large array, $M - 1$ is very close to M, thereby indicating that the RSM method is comparable with the classical MUSIC method in DOA estimation performance. This claim will be proved by simulations in Section V. Meanwhile, the RSM method obtains the initial information for the refinement procedure by using the new observation data and the reference signal, which is capable of capturing the signal information while efficiently suppressing the additive noise. Thereby, the RSM method is more accurate than the linear operation based methods for bearing estimation, especially in some severe environments such as small the sample size and/or low signal-to-noise ratio (SNR). Furthermore, the RSM method finds the signal subspace by the successive refinement procedure with p recursions, avoiding the estimated covariance matrix and its eigendecomposition. Therefore, the RSM method is much more computationally efficient than the classical MUSIC estimator that relies on the estimated covariance matrix and its EVD computation.

3.2 Computational Complexity Requirement

From (6)-(8), we can see that the dominate computational cost among them is the calculation of the matched filter, which requires $M - 1$ complex multiplications and $M - 2$ additions for each snapshots, equivalently approximately $M - 1$ floating point operations (or flops), and thereby $O((M - 1)N)$ flops for each matched filter. Consequently, the RSM method needs $O(p(M - 1)N)$ flops for calculating the signal subspace. However, to find the signal or noise subspace, the EVD-based MUSIC method resorts to the estimated covariance matrix and its EVD computation, requiring around $O(M^2 N + M^3)$ flops. Note that $O(M^2 N + M^3) \gg O(p(M - 1)N)$ as $M \gg p$. Therefore, the RSM method is much more computationally efficient than EVD-based MUSIC estimator, in particular when M is large.

4. Extension to Coherent Signal Case

In the presence of coherent signals, the signal covariance matrix becomes singular, which leads to the divergence of some signal vectors into a noise subspace, and thereby a signal subspace cannot be obtained correctly. To cure this problem, we apply the forward/backward spatial smoothing (FBSS) technique [10] to the sensor data. Using the FBSS method, we obtain the $q \times 1$ sensor data vector of the lth forward subarray:

$$\begin{aligned} \boldsymbol{y}_l^f(t) &= [y_l(t), y_{l+1}(t), \cdots, y_{q+l-1}]^T \\ &= \boldsymbol{A}_q(\boldsymbol{\theta}) \boldsymbol{D}^{l-1} \boldsymbol{u}(t) + \boldsymbol{n}_l(t), \ 2 \leq l \leq L, \end{aligned} \tag{10}$$

where $\boldsymbol{A}_q(\boldsymbol{\theta}) = [\boldsymbol{a}_q(\theta_1), \boldsymbol{a}_q(\theta_2), \cdots, \boldsymbol{a}_q(\theta_p)]$ with $\boldsymbol{a}_q(\theta_i) = \left[1, e^{j w_0 \tau_1(\theta_i)}, \cdots, e^{j w_0 \tau_{q-1}(\theta_i)}\right]^T$, $\boldsymbol{n}_l(t) = [n_l(t), n_{l+1}(t), \cdots, n_{q+l-1}(t)]^T$, and $L = M - q + 1$ is the number of the forward subarrays. In the sequel, the $q \times (L - 1)$ forward smoothed matrix can be written as

$$\bar{\boldsymbol{Y}}^f(t) = \frac{1}{\sqrt{L-1}} \left[\boldsymbol{y}_2^f(t), \boldsymbol{y}_3^f(t), \cdots, \boldsymbol{y}_L^f(t) \right]. \tag{11}$$

Accordingly, the lth forward smoothed reference signal is formed by $d_l^f(t) = y_1(t) (l = 2, 3, \cdots, L)$, and thereby the $(L-1) \times 1$ forward smoothed reference signal vector can be expressed as

$$\bar{\boldsymbol{d}}^f(t) = \frac{1}{\sqrt{L-1}} [y_1(t), y_1(t), \cdots, y_1(t)]^T. \tag{12}$$

Similarly, we can also get the $q \times 1$ sensor data vector of the lth backward subarray:

$$\begin{aligned} \boldsymbol{y}_l^b(t) &= [y_{M-l+2}(t), y_{M-l+1}(t), \cdots, y_{L-l+2}]^H \\ &= \boldsymbol{A}_q(\boldsymbol{\theta}) \boldsymbol{D}^{l-2} \left(\boldsymbol{D}^{q-1} \boldsymbol{u}(t) \right)^* + \boldsymbol{n}_{M-l+2}^*(t), 2 \leq l \leq L. \end{aligned} \tag{13}$$

Consequently, the $q \times (L - 1)$ backward smoothed matrix and the $q \times 1$ backward smoothed reference signal vector may be written as:

$$\bar{\boldsymbol{Y}}^b(t) = \frac{1}{\sqrt{L-1}} \left[\boldsymbol{y}_2^b(t), \boldsymbol{y}_3^b(t), \cdots, \boldsymbol{y}_L^b(t) \right] \tag{14}$$

and

$$\bar{\boldsymbol{d}}^b(t) = \frac{1}{\sqrt{L-1}} [y_1^*(t), y_1^*(t), \cdots, y_1^*(t)]^T. \tag{15}$$

Thus, it follows from (12) and (15) that the forward/backward smoothed sensor data matrix can be formed by

$$\bar{\boldsymbol{Y}}_0(t) = \frac{1}{\sqrt{2}} \left[\bar{\boldsymbol{Y}}^f(t), \bar{\boldsymbol{Y}}^b(t) \right]. \tag{16}$$

Accordingly, the forward/backward smoothed reference signal vector is given as

$$\bar{\boldsymbol{d}}_0(t) = \frac{1}{\sqrt{2}} \left[\bar{\boldsymbol{d}}^{f,T}(t), \bar{\boldsymbol{d}}^{b,T}(t) \right]^T. \quad (17)$$

With the smoothed observation data and reference signal, the initial information for the refinement procedure can be calculated as:

$$\begin{aligned} \boldsymbol{r}_{\bar{y}_0 \bar{d}_0} &= E\left[\bar{\boldsymbol{y}}_0(t)\bar{\boldsymbol{d}}_0^*(t)\right] \\ &= \frac{1}{2}\left(E\left[\bar{\boldsymbol{Y}}^f(t)\bar{\boldsymbol{d}}^{f,*}(t)\right] + E\left[\bar{\boldsymbol{Y}}^b(t)\bar{\boldsymbol{d}}^{b,*}(t)\right]\right) \\ &\triangleq \frac{1}{2}\left(\boldsymbol{r}^f_{\bar{y}\bar{d}} + \boldsymbol{r}^b_{\bar{y}\bar{d}}\right), \end{aligned} \quad (18)$$

where

$$\begin{aligned} \boldsymbol{r}^f_{\bar{y}\bar{d}} &= E\left[\bar{\boldsymbol{Y}}^f(t)\bar{\boldsymbol{d}}^{f,*}(t)\right] \\ &= \frac{1}{L-1}\sum_{l=2}^{L}\left[r_{l,1}, \cdots, r_{l+q-1,1}\right]^T \end{aligned} \quad (19)$$

$$\begin{aligned} \boldsymbol{r}^b_{\bar{y}\bar{d}} &= E\left[\bar{\boldsymbol{Y}}^b(t)\bar{\boldsymbol{d}}^{b,*}(t)\right] \\ &= \frac{1}{L-1}\sum_{l=2}^{L}\left[r^*_{M-l+2,1}, \cdots, r^*_{L-l+2,1}\right]^T \\ &= \boldsymbol{J}_q \boldsymbol{r}^{f,*}_{\bar{y}\bar{d}}, \end{aligned} \quad (20)$$

in which $\boldsymbol{J}_q$ is a $q \times q$ reversal matrix with 1 along the cross-diagonal and zero elsewhere, and $r_{i,1} (i = 2, 3, \cdots, M)$ are defined in (9). In the sequel, the cross-correlation between $\bar{\boldsymbol{Y}}_0(t)$ and $\bar{\boldsymbol{d}}_0(t)$ can be eventually calculated as

$$\boldsymbol{r}_{\bar{y}_0 \bar{d}_0} = \frac{1}{2}\left(\boldsymbol{r}^f_{\bar{y}\bar{d}} + \boldsymbol{J}_q \boldsymbol{r}^{f,*}_{\bar{y}\bar{d}}\right). \quad (21)$$

It follows from (20)-(22) that the additive noise has been efficiently eliminated. Therefore, similar to the refinement procedure presented in Subsection A, we employ the normalized cross-correlation, namely the matched filter $\bar{\boldsymbol{h}}_1 = \boldsymbol{r}_{\bar{y}_0 \bar{d}_0}/\|\boldsymbol{r}_{\bar{y}_0 \bar{d}_0}\|_2$, as the initial information to partition the smoothed observation data into a signal subspace.

The RSM algorithm for the bearing estimation of coherent signals is summarized as follows:

Step1: Apply the FBSS technique to the $(M-1) \times 1$ sensor data $\boldsymbol{y}_0$ and obtain the forward smoothed sensor data matrix $\bar{\boldsymbol{Y}}^f_0(t)$ in (12). Meanwhile, the forward smoothed reference signal vector $\bar{\boldsymbol{d}}^f_0(t)$ is attained by (13).

Step2: Perform the following p recursions:

For $i = 1, 2, \cdots, p$:

$\hat{\boldsymbol{r}}^f_{\bar{y}_{i-1}\bar{d}_{i-1}} = \frac{1}{N}\sum_{t=1}^{N} \bar{\boldsymbol{Y}}^f_{i-1}(t)\bar{\boldsymbol{d}}^{f,*}_{i-1}(t)$,

$\hat{\boldsymbol{r}}_{\bar{y}_{i-1}\bar{d}_{i-1}} = \frac{1}{2}(\hat{\boldsymbol{r}}^f_{\bar{y}_{i-1}\bar{d}_{i-1}} + \boldsymbol{J}_q \hat{\boldsymbol{r}}^{f,*}_{\bar{y}_{i-1}\bar{d}_{i-1}})$,

$\hat{\boldsymbol{h}}_i = \hat{\boldsymbol{r}}_{\bar{y}_{i-1}\bar{d}_{i-1}}/\|\hat{\boldsymbol{r}}_{\bar{y}_{i-1}\bar{d}_{i-1}}\|_2$,

$\bar{\boldsymbol{d}}^f_i(t) = \hat{\boldsymbol{h}}^H_i \bar{\boldsymbol{Y}}^f_{i-1}(t)$,

$\bar{\boldsymbol{Y}}^f_i(t) = \bar{\boldsymbol{Y}}^f_{i-1}(t) - \hat{\boldsymbol{h}}_i \bar{\boldsymbol{d}}^f_i(t)$.

Attain the estimated signal subspace $\hat{\boldsymbol{T}}_s = \left[\hat{\boldsymbol{h}}_1, \cdots, \hat{\boldsymbol{h}}_p\right]$.

Step3: Obtain the bearing estimates of the coherent signals by searching the highest p peaks of the spatial spectrum $S(\theta) = \frac{1}{\boldsymbol{a}_q(\theta)\hat{\boldsymbol{P}}_n \boldsymbol{a}_q(\theta)}$, where $\hat{\boldsymbol{P}}_n = \boldsymbol{I}_q - \hat{\boldsymbol{T}}_s(\hat{\boldsymbol{T}}^H_s \hat{\boldsymbol{T}}_s)^{-1}\hat{\boldsymbol{T}}^H_s$. Alternatively, the bearings can also be yielded by the root-MUSIC estimator: finding the p roots, say $z_1, z_2, \cdots, z_p$ that have the largest magnitude, of the root-MUSIC polynomial $s(z) = z^{q-1}\boldsymbol{p}^H(z)\hat{\boldsymbol{P}}_n \boldsymbol{p}(z)$ where $\boldsymbol{p} = \left[1, z. \cdots, z^{q-1}\right]^T$, yields the bearing estimates as $\theta_i = \arcsin\left(\frac{c\arg(z_i)}{w_0 d}\right)$ in which $\arg(z_i)$ denotes the phase angle of the complex number z_i.

Since the first sensor data $y_1(t)$ cannot be used in the smoothed observation data, the number of subarrays is reduced from L to $L-1$ in the forward/backward spatial smoothing procedures. However, the RSM method is still comparable with the MUSIC estimator based on the FBSS with L subarrays and the EVD technique because it uses the cross-correlation between the smoothed observation data and the smoothed reference signal as the initial information for the refinement procedure, which efficiently eliminates the additive noise while capturing the signal information.

From the Step 2 of the summarized algorithm for the bearing estimations of coherent signals, we can observe that the RSM method needs around $O(pq(L-1)N)$ flops to find the signal subspace. Note that the FBSS-based MUSIC estimator requires about $O(q^2LN + q^3)$ flops for the same purpose. We generally let $q = M - p$ and thereby $L = M - q + 1 = p + 1$. As a result, as $M \gg p$ we attain $O(p^2(M-p)N) \approx O(p^2MN)$ flops for the RSM and $O((M-p)^2(p+1)N + (M-p)^3) \approx O((p+1)M^2N + M^3)$ flops for the FBSS-based MUSIC. Therefore, the RSM is more computationally efficient than the FBSS-based MUSIC in the calculation of the signal or noise subspace, especially as M becomes large.

5. Numerical Results

We now evaluate the performance of the RSM method for bearing estimation by computer simulations. Since the SUMWE method [6] outperforms the other linear operation based methods, such as the BEWE [3], the PM [4] and the SWEDE [5], but performs worse than the FBSS-based MUSIC estimator in the bearing estimation accuracy, we will compare the performance of the RSM method with that of the SUMWE and FBSS-based methods.

Let the number of sensors of the ULA be 10. The spacings between the adjacent sensors equal half-wavelength. Suppose that there are two signals impinging upon the ULA consisting of 10 sensors from the same signal source. The first is a direct-path signal and the second refers to the scaled and delayed replica of the first signal that represents the multipath or the "smart" jammer. The propagation constants are $\{1, 0.5+j0.3\}$. The true bearings are assumed to be $\{2^o, 7^o\}$. The number of signals is assumed to be known *priori* or estimated by the method [7]. Meanwhile, the background noise is a stationary Gaussian white random process that is uncorrelated with the signals. The SNR is defined as the ratio of the variance of signal to the variance of noise.

We have run 1000 independent trials to calculate the root-mean-square errors (RMSEs) of the estimated bearings for the RSM, the EVD-based MUSIC, and the SUMWE approaches. The RMSEs of the estimated bearings versus SNR are plotted in Fig. 1, which indicates that the RSM method is much more accurate than the SUMWE method in bearing estimation as SNR<5dB, and more accurate than the FBSS-based MUSIC estimator when SNR is lower than 0dB. As SNR becomes higher than 5dB, the RSM method, the SUMWE approach and the FBSS-based MUSIC estimator with 2 subarrays nearly obtain the same estimation accuracy. Since the RSM method only has 2 subarrays, its estimation accuracy is a little lower than that of the FBSS-based MUSIC estimator with 3 subarrays, but still outperforms the FBSS-based MUSIC method when SNR is low.

Fig. 2 shows the RMSEs of the estimated bearings versus the number of snapshots. From Fig. 2, it can be observed that the estimation accuracy of the RSM method is very close to that of the FBSS-based MUSIC estimator. Meanwhile, the RSM method has a higher estimation accuracy than the SUMWE method as $N < 64$ and has the estimation accuracy closed to that of the SUMWE estimator when $N \geq 64$.

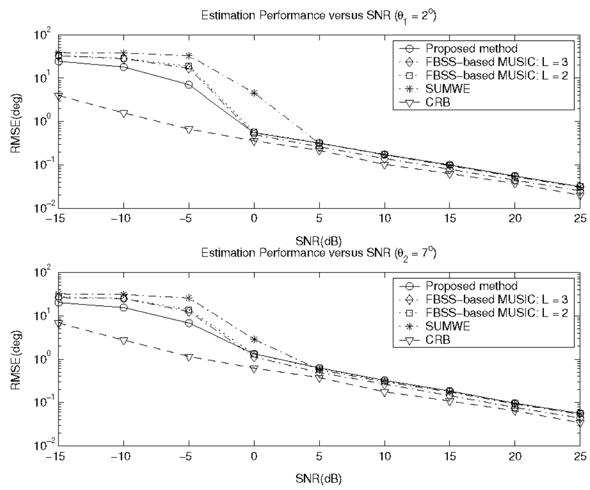

Fig. 1 RMSEs of estimated DOAs of coherent signals versus SNR. DOAs of signal 1 and 2 are 2^o and 7^o. $M = 10$, $q = 8$ and $N = 128$.

6. Conclusions

We have developed a recursion method for bearing estimation in this paper. The RSM method finds the *signal subspace* by means of a successive refinement procedure, and avoids the estimate of the covariance matrix and the EVD computation, thereby requiring much lower computational complexity than the EVD-based MUSIC estimator, in particular for a large array.

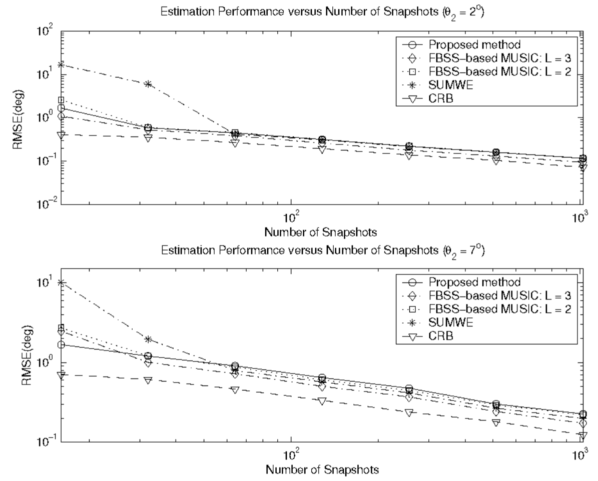

Fig. 2 RMSEs of estimated DOAs of coherent signals versus number of snapshots. DOAs of signal 1 and 2 are 2^o and 7^o. $M = 10$, $q = 8$ and SNR=5dB.

References

[1] P. Stoica and K. C. Sharman, "Maximum likelihood methods for direction-of-arrival estimation," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 38, pp. 1132-1143, July 1990.

[2] R. O. Schmidt. *A Signal Subspace Approach To Multiple Emiiter Location Spectral Estimation.* Ph. D. thesis. Stanford University, Stanford, CA, Nov. 1981.

[3] P. Stoica and T. Söerström, "Statistical analysis of a subspace method for bearing estimation without eigendecomposition," *Proc. Inst. Elect. Eng. F*, vol. 139, no. 4, pp. 301-305, 1992.

[4] A. Eriksson, P. Stoica, and T. Söerström, "On-line subspace algorithms for tracking moving sources," *IEEE Trans. Signal Processing*, vol. 42, pp. 2319-2330, Sept. 1994.

[5] S. Marcos, A. Marsal, and M. Benidir, "The propagator method for source bearing estimation," *Signal Process.*, vol. 42, no. 2, pp. 121-138, 1995.

[6] J. Xin and A. Sano, "Computationally efficient subspace-based method for direction-of-arrival estimation without eigendecomposition," *IEEE Trans. Signal Processing*, vol. 52, no. 4, pp. 876-893, Apr. 2004.

[7] L. Huang, S. Wu and X. Li, "Reduced-rank MDL method for source enumeration in high-resolution array processing," to appear in *IEEE Trans. Signal Processing*, November, 2007.

[8] J. S. Goldstein, I. S. Reed, and L. L. Scharf, "A multistage representation of the wiener filter based on orthogonal projections," *IEEE Trans. Information Theory*, vol. 44, no. 7, pp. 2943-2959, Nov. 1998.

[9] L. Huang, S. Wu, D. Feng and L. Zhang, "Low complexity method for signal subspace fitting," *Electronics Letters*, vol. 40, pp. 847-848, Jul. 2004.

[10] S. U. Pillai and B. H. Kwon, "Forward/backward spatial smoothing techniques for coherent signal identification," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, no. 1, pp. 8-15, January 1989.