Estimation of the Number of Sources by Using an MMSE-based MDL Criterion without Eigendecomposition

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Abstract

A new computationally simple MDL approach is addressed in this paper. Unlike the eigenvalue-based MDL methods, the proposed method suggests to use the minimum mean square errors (MMSEs) of the multi-stage Wiener filter (MSWF) to calculate the description length required to encode the observed data. As a result, the proposed method is more robust to the nonuniform noise than the eigenvalue-based MDL methods. On the other hand, since the proposed method does not involve the estimation of the observed covariance matrix and its eigedecomposition, its computational complexity can be significantly reduced. Numerical results are presented to illustrate the consistency and robustness of the proposed method.

1. Introduction

Computationally efficient and robust methods for source detection are of significant interest in practical applications of array processing [1]-[10]. This is due to the fact that, on one hand, when a large array is employed to localize the signals of interest (SOI) in a real-time manner, the required computational load of the classical methods is quite heavy. On the other hand, the assumption of spatially and temporally white noise across such a large array might not be true because the unknown noise environment may change slowly with time [11], and the sensor noises thereby become correlated from sensor to sensor and unequal in power level. Although the sensor noises may be uncorrelated among all sensors in many practical applications, their power levels are in general unequal due to the nonidealities of the practical arrays, such as the nonideality of the receiving channel, the nonuniformity of the sensor response and the mutual coupling between sensors. As a consequence, the sensor noise becomes a spatially inhomogeneous white process, $i.e.$, of unequal power level and uncorrelated from sensor to sensor. When implemented in such an environment, the classical model-dependent methods, such as the classical MDL methods [1], may fail to yield the reliable estimate of the number of sources in a real-time manner.

While there have been some papers, such as [4]-[8], dealing with the robust estimation of the number of sources, these methods need to be further improved in computational complexity and/or detection performance. The eigenvector-based methods, such as [8], can yield the reliable estimate of the number of sources in the nonuniform noise environment. Similar to the eigenvalue-based methods [1], however, the eigenvector-based methods necessarily involve the estimation of the observed covariance matrix and its EVD calculation, making them to be quite computationally intensive. Although the MDL method addressed by Fishler and Poor [7] is robust against the deviations from the assumption of spatially and temporally white noise, it involves N iterations and each iteration needs the EVD computation, thereby requiring around $O(N^4)$ flops besides the calculation of the covariance matrix, which is rather computationally burdensome especially when N becomes large. Recently, we addressed a computationally efficient Gerschgorin disk estimator for source number without eigendecomposition (GDEWE) in [6]. The GDEWE method is more robust and computationally efficient than the classical methods for source enumeration. Nevertheless, like the GDE estimator, the detection performance of the GDEWE estimator also relies on a non-increasing function that needs to be carefully designed in the practical applications.

In this paper, a novel computationally efficient MDL method is addressed for the detection of source number. In contrast to the eigenvalue-based MDL methods, the proposed mMDL method only involves the MMSEs of the MSWF [14] to calculate the code length of the observed data, independent of the eigenvalues of the observed covariance matrix. As a result, the mMDL method is more robust to nonuniform noise than the eigenvalue-based MDL methods. Meanwhile, the proposed method does not involve the estimation of the observed covariance matrix and its EVD computation, requiring lower computational cost than the EVD-based methods, particularly for a large array.

2. Data Model

Consider an array of N sensors receiving q $(q < N - 1)$ narrow-band far-field sources from distinct directions $\theta_1, \cdots, \theta_q$. For simplicity, we assume that the array and the

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sources are in the same plane. In the sequel, the ℓth snapshot vector consisting of the sensor array outputs, excluding the last sensor output, can be written as

$$\boldsymbol{x}(t_\ell)=[x_1(t_\ell),\cdots,x_M(t_\ell)]^T=\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t_\ell)+\boldsymbol{n}(t_\ell), \quad (1)$$

where $M=N-1$, $(\cdot)^T$ denotes the transpose operation and

$$\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1),\cdots,\boldsymbol{a}(\theta_q)] \quad M\times q \text{ steering matrix};$$
$$\boldsymbol{s}(t_\ell) = [s_1(t_\ell),\cdots,s_q(t_\ell)]^T \; q\times 1 \text{ source waveform vector};$$
$$\boldsymbol{n}(t_\ell) = [n_1(t_\ell),\cdots,n_M(t_\ell)]^T \; M\times 1 \text{ sensor noise vector}.$$

For a uniform linear array (ULA) with inter-sensor spacing d, the steering vector may be given as $\boldsymbol{a}(\theta_i)=\left[1,e^{j2\pi d\sin(\theta_i)/\lambda},\cdots,e^{j2\pi d(M-1)\sin(\theta_i)/\lambda}\right]^T$ $(i=1,\cdots,q)$ where λ denotes the wavelength and q denotes the *unknown* number of sources. The source waveform $s_i(t_\ell)$ $(i=1,\cdots,q)$ is assumed to be a jointly stationary, statistically uncorrelated, zero-mean complex Gaussian random process. Meanwhile, the additive noise $\boldsymbol{n}(t_\ell)$ is assumed to be an ergodic, zero-mean, spatially and temporally white complex Gaussian process with the covariance matrix $\sigma_n^2\boldsymbol{I}_M$. In addition, the sensor noise is presumed to be uncorrelated with the sources.

Under these assumptions, the observed data $\boldsymbol{x}(t_\ell)$ is a complex Gaussian random process with zero mean and the covariance matrix:

$$\boldsymbol{R}_x = E[\boldsymbol{x}(t_\ell)\boldsymbol{x}^H(t_\ell)] = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{R}_s\boldsymbol{A}^H(\boldsymbol{\theta})+\sigma_n^2\boldsymbol{I}_M, \quad (2)$$

where $E[\cdot]$ represents expectation, $(\cdot)^H$ is the Hermitian transpose and $\boldsymbol{R}_s = E[\boldsymbol{s}(t_\ell)\boldsymbol{s}^H(t_\ell)]$ denotes the signal covariance matrix. In the practical applications, however, only the finite number of snapshots is available. In the sequel, the sample-covariance matrix is calculated by $\hat{\boldsymbol{R}}_x = (1/L)\sum_{\ell=1}^{L}\boldsymbol{x}(t_\ell)\boldsymbol{x}^H(t_\ell)$.

3. Proposed MMSE-Based MDL Estimator

It is shown in [1] that, for a given data set and a family of probabilistic models, the MDL principle is to select the model that offers the shortest description length of the data. Specifically, given an observed data set $\boldsymbol{X}=\{\boldsymbol{x}(t_\ell)\}_{\ell=1}^{L}$ and a probabilistic model $p(\boldsymbol{X}|\boldsymbol{\Theta})$, where $\boldsymbol{\Theta}$ denotes an unknown parameter vector of the model, the shortest description length required to encode the data using the model can be asymptotically written as

$$\mathcal{L}\{\boldsymbol{x}(t_\ell)\} = -\log p(\boldsymbol{X}|\hat{\boldsymbol{\Theta}})+\frac{1}{2}K\log L, \quad (3)$$

where $\hat{\boldsymbol{\Theta}}$ is the maximum likelihood (ML) estimate of $\boldsymbol{\Theta}$ and K denotes the number of free parameters in $\hat{\boldsymbol{\Theta}}$. Since the observed data $\{\boldsymbol{x}(t_\ell)\}$ are assumed to be statistically independent complex Gaussian random vectors with zeros mean, their joint probability density can be given by

$$p(\boldsymbol{X}|\boldsymbol{\Theta}) = \prod_{\ell=1}^{L}\frac{1}{\pi^q\|\boldsymbol{R}_x\|}\exp\left\{-\boldsymbol{x}(t_\ell)^H\boldsymbol{R}_x^{-1}\boldsymbol{x}(t_\ell)\right\}, (4)$$

where $\|\cdot\|$ is the determinant operation. Taking the logarithm of (4) and omitting the terms independent of $\boldsymbol{\Theta}$, we define the negative log-likelihood function as

$$\mathcal{F}(\boldsymbol{\Theta}) = L\log\|\boldsymbol{R}_x\| + \mathrm{tr}\left(\boldsymbol{R}_x^{-1}\hat{\boldsymbol{R}}_x\right). \quad (5)$$

To proceed to the derivation of the MMSE-based MDL method, we need to use the following results of the MSWF. The MSWF is given in Appendix A.

Lemma 1: The determinant of the observed covariance matrix $\boldsymbol{R}_{x_0}$ is equal to that of $\boldsymbol{R}_e$, the covariance matrix of the error of the MSWF, *i.e.*,

$$\|\boldsymbol{R}_{x_0}\| = \|\boldsymbol{R}_e\| = \prod_{i=1}^{M}\rho_i, \quad (6)$$

where $\boldsymbol{R}_{x_0} \triangleq E[\boldsymbol{x}_0(t_\ell)\boldsymbol{x}_0^H(t_\ell)]$, $\boldsymbol{R}_e \triangleq \mathrm{diag}\left([\rho_1,\cdots,\rho_M]^T\right)$, and $\rho_i \triangleq E[|e_i(t_\ell)|^2]$ $(i=1,\cdots,M)$ is the MMSE at the ith stage of the MSWF.

Proof: The proof of Lemma 1 is easily completed by following the results in [12]-[14], and omitted here due to the space limit. □

Lemma 2: The MMSEs of the MSWF ρ_i $(i=1,\cdots,M)$ satisfy

$$\rho_1 \geq \cdots \geq \rho_q > \rho_{q+1} = \cdots = \rho_M = \sigma_n^2, \quad (7)$$

where σ_n^2 is the noise variance and q denotes the true number of sources.

Proof: It is easy to prove Lemma 2 by using the results of [6] and [10]. The proof of Lemma 2 is omitted due to the space limit. □

In the sequel, updating the observed covariance matrix $\boldsymbol{R}_x$ by $\boldsymbol{R}_{x_0}$, substituting (6) and (7) into (5), and assuming that k is the *supposed* number of sources, we obtain

$$\mathcal{F}(\boldsymbol{\Theta}) = L\log\left(\prod_{i=1}^{k}\rho_i\times\prod_{i=k+1}^{M}\sigma_n^2\right)+\mathrm{tr}\left(\boldsymbol{R}_{x_0}^{-1}\hat{\boldsymbol{R}}_{x_0}\right). (8)$$

On the other hand, it follows from Appendix A that the free parameter vector can be given by

$$\boldsymbol{\Theta}^T = \left(\rho_1,\cdots,\rho_k,\sigma_n^2,w_1,\cdots,w_k,\boldsymbol{h}_1,\cdots,\boldsymbol{h}_k\right), \quad (9)$$

where w_i $(i=1,\cdots,k)$ are the scalar Wiener Filters in the backward recursion of the MSWF and $\boldsymbol{h}_i$ $(i=1,\cdots,k)$ are the matched filters in the forward recursion of the MSWF. Actually, not all the free parameters are independent of each other. It follows from the MSWF given in Appendix A that both ρ_i and w_i rely on the desired signals $d_i(t_\ell)$ which are obtained by filtering the observed data with the matched filter $\boldsymbol{h}_i$, $i.e.$, $d_i(t_\ell) = \boldsymbol{h}_i^H \boldsymbol{x}_0(t_\ell)$. In the sequel, the parameter vector can be finally reduced to be

$$\boldsymbol{\Theta}^T = \left(\sigma_n^2, \boldsymbol{h}_1, \cdots, \boldsymbol{h}_k\right). \tag{10}$$

On the other hand, notice that the matched filters $\boldsymbol{h}_i$ $(i = 1, \cdots, k)$ are the orthogonal and normalized vectors, which lead to a reduction of $(2k+2(1/2)k(k-1))$ in the free parameter number. As a result, the number of free parameters in $\boldsymbol{\Theta}$ can be counted as

$$K = 2Mk+1-2k-2(1/2)k(k-1) = k(2M-k-1)+1. \tag{11}$$

It is easy to show that the estimated MMSE yielded by the backward recursion of the MSWF, $\hat{\rho}_i = \hat{\sigma}_{d_i}^2 - |\hat{\delta}_{i+1}|^2/\hat{\rho}_{i+1}$, is the ML estimate of ρ_i. Consequently, it follows from (7) that the ML estimate of σ_n^2 can be given by

$$\hat{\sigma}_n^2 = \frac{1}{M-k}\sum_{i=k+1}^{M} \hat{\rho}_i. \tag{12}$$

Therefore, substituting the ML estimates of ρ_i $(i = 1, \cdots, k)$, σ_n^2 and $\boldsymbol{R}_{x_0}$ into (8) and (3), omitting the constant terms and applying the same argument used in [1] yields

$$\begin{aligned}\mathcal{L}\{\boldsymbol{x}(t_\ell)\} &= L\log\left(\prod_{i=1}^{k}\hat{\rho}_i \times \prod_{i=k+1}^{M}\hat{\sigma}_n^2\right) + \frac{1}{2}k\left(2M-k-1\right)\log L \\ &\triangleq \mathcal{F}(k) + \mathcal{P}(k), \end{aligned}\tag{13}$$

where

$$\mathcal{F}(k) = L\left(M-k\right)\log\left(\frac{1/\left(M-k\right)\sum_{i=k+1}^{M}\hat{\rho}_i}{\left(\prod_{i=k+1}^{M}\hat{\rho}_i\right)^{1/(M-k)}}\right) \tag{14}$$

$$\mathcal{P}(k) = \frac{1}{2}\left(k(2M-k-1)\right)\log L, \tag{15}$$

are the log-likelihood function and the penalty function, respectively. Thus, the number of sources can be yielded by minimizing the MMSE-based MDL (mMDL) criterion:

$$\hat{q} = \arg\min_{k=0,\cdots,M-1} \text{mMDL}(k), \tag{16}$$

where $\text{mMDL}(k) = \mathcal{F}(k) + \mathcal{P}(k)$.

Remarks: As noted in Section I, the EVD-based MDL methods [1] necessarily involve the estimate of the covariance matrix and its EVD computation, which require around $O(N^2L)+O(N^3)$ flops. To correctly detect the sources, the rMDL method [7] do not terminate the iterative procedure until a stationary point is reached, generally requiring N iterations, and each iteration includes the EVD computation of an updated covariance matrix. As a result, the rMDL method needs around $O(N^4)$ flops besides the calculation of the observed covariance matrix that also requires $O(N^2L)$ flops. However, in the proposed mMDL method, the MMSEs of the MSWF can be directly yielded from the MSWF, avoiding the estimate of the covariance matrix and its EVD computation. Meanwhile, note that the forward recursion procedure only involves complex vector-vector products, and does not include any complex matrix-vector products, thereby requiring around $O(M)(M = N-1)$ flops for each snapshot and each stage. In the sequel, after performing N forward recursions, the required computational cost is only about $O(N^2L)$ flops. Meanwhile, it should be noted that the backward recursion only involves complex scalar-scalar products that are fiddling in computational complexity compared to the complex vector-vector products. Therefore, the mMDL method requires much less computational cost than the EVD-based MDL methods [1], [7].

4. Numerical Results

Assume that there exist 2 narrow-band sources impinging upon a ULA with 10 sensors. Meanwhile, the inter-sensor spacing is equal to half wavelength. We first consider the scenario where the sensor noise is a stationary, spatially and temporally white, Gaussian random process that is uncorrelated with the sources. In this case, the SNR is defined as the ratio of the power of signals to the power of noise at each sensor. Five hundred independent trials have been run to calculate the empirical probabilities of correct detection of the mMDL, cMDL and rMDL methods. Fig. 1 demonstrates the empirical results of the three MDL methods versus the number of snapshots. Observed that as the number of snapshots tends to infinity, the proposed mMDL method, the cMDL method [1] and the rMDL method [7] converge to one in probability of correct detection. This thereby indicates that all the MDL methods are consistent for the case of spatially and temporally white noise. Nevertheless, when the number of snapshots is less than 200, neither the mMDL nor the rMDL method [7] is as accurate as the cMDL method [1]. As addressed in [7], the rMDL method is less accurate than the cMDL method because it ignores the a priori knowledge that the sensor noise is spatially and temporally white. Note that the mMDL method only employs 9 sensor outputs as the observed data to calculate the MMSEs of the MSWF, reducing the aperture of the array from 10 to 9, and is thereby less accurate than the cMDL and rMDL methods as the number of snapshots varies from 60 to 200. When $L \leq 60$, however, the proposed mMDL method surpasses the rMDL method and is very close to cMDL method in detection performance. Fig. 2 shows the empirical probability of correct detection versus the angle separation. Again, due to the reduced aperture of the array, the proposed mMDL method only yields the lower accurate estimation of the source number than the cMDL method when the angle separation varies from $3°$ to $5.5°$, and is as accurate as the latter when the angle separation becomes larger. While the rMDL method is a little more accurate than the mMDL method for small angle separation, it needs 10 iterations and each iteration involves the EVD computation of the updated

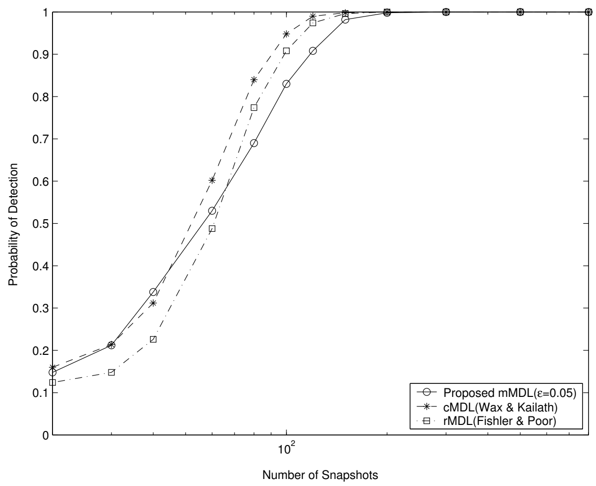

Figure 1. Probability of correct detection versus number of snapshots for the case of spatially white noise. $[\theta_1, \theta_2] = [2.5°, 7.8°]$, SNR $= -3$dB, $N = 10$.

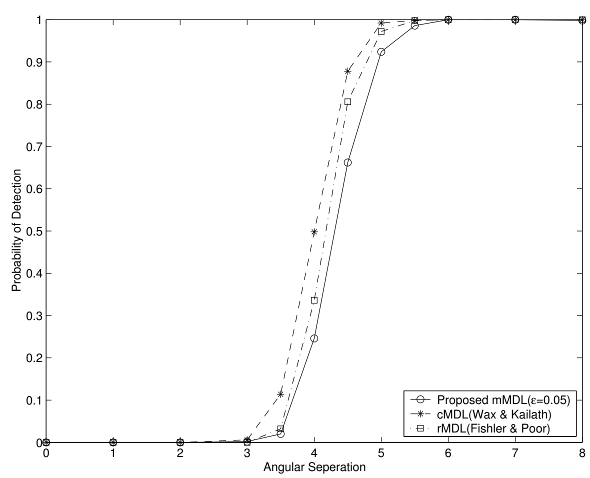

Figure 2. Probability of correct detection versus angular separation for the case of spatially white noise. $[\theta_1, \theta_2] = [2.5°, 2.5° + \Delta\theta]$, SNR $= -3$dB, $L = 150$, and $N = 10$.

covariance matrix, thereby requiring about $O(10^4)$ flops besides the calculation of the observed covariance matrix that still needs $O(100L)$ flops. However, the mMDL method only needs around $O(81L)$ flops which is much less than that of the rMDL method. Therefore, the small loss of the mMDL method in detection performance can be balanced by its computational simplicity.

To examine the robustness of the mMDL method, we have performed five hundred independent trials to calculate the probability of correct detection for the case of nonuniform noise. Similar to [7], the noise power level is taken as

$$\sigma_n^2 \boldsymbol{I}_N + (\sigma_n^2/2)\mathrm{diag}([-0.9, -0.7, -0.5, -0.3, -0.1, 0.1, 0.3, 0.5, 0.7, 0.9])$$

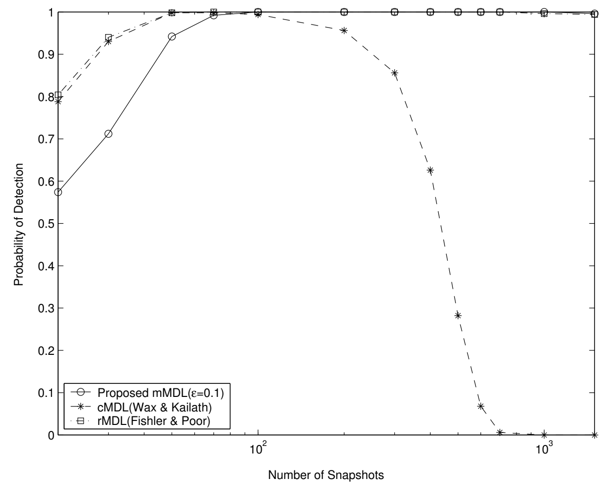

Figure 3. Probability of correct detection versus number of snapshots for the case of nonuniform noise. $[\theta_1, \theta_2] = [2.5°, 7.8°]$, SNR $= 0$dB, $N = 10$.

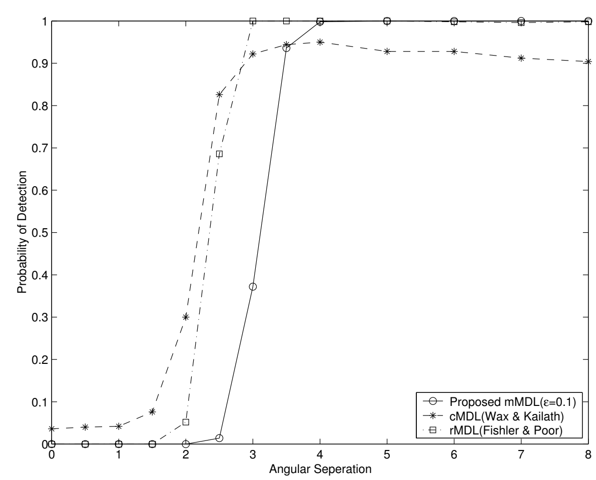

Figure 4. Probability of correct detection versus angular separation for the case of nonuniform noise. $[\theta_1, \theta_2] = [2.5°, 2.5° + \Delta\theta]$, SNR $= 0$dB, $N = 250$, and $N = 10$.

to simulate the deviations of up to 3dB from the normal noise power level σ_n^2. In this scenario, the SNR is defined as the ratio of the power of signals to the *averaged* power of noises. Fig. 3 depicts the empirical probabilities of correct detection varying with the number of snapshots. It is implied in Fig. 3 that both the mMDL method and the rMDL method can correctly detect the sources as the number of snapshots increases infinitely. The eigenvalue-based cMDL method, however, fails to correctly enumerate the sources as the number of snapshots is greater than 100. Moreover, as the number of snapshots increases, the empirical probability of correct detection of the cMDL method converges to zero. This thereby implies that both the mMDL method and the rMDL method offer the robustness whereas the cMDL method is of non-robustness in this scenario. To demonstrate the detection performance of the MDL methods for different angle separation, we plotted the empirical probability of correct detection varying with the angle separation in Fig. 4. Similarly, the cMDL method fails to correctly detection the sources no matter how large the angle separation is. Nevertheless, since the mMDL method only uses the MMSEs of the MSWF instead of the eigenvalues, it is more robust against the deviations than the eigenvalue-based cMDL method. Thus, the mMDL method is superior to the rMDL method in computational complexity and outperforms the cMDL method in robustness and computational complexity.

5. Conclusions

We have addressed an MMSE-based MDL method for source number estimation in this paper. Since the proposed mMDL method only involves the forward and backward recursions of the MSWF, and avoids the estimation of the observed covariance matrix and its EVD calculation, giving it the advantage of computational simplicity. Additionally, since the unequal noise power levels at the sensors only make the smallest eigenvalues to be significantly unequal but scarcely affect the smallest MMSEs of the MSWF, the proposed MDL method is more robust against the deviations from the assumption of spatially and temporally white noise than the eigenvalue-based MDL methods.

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Appendix A. The MSWF

The full-rank MSWF is presented as follow:

Initialization:

$d_0(t_\ell) = x_{M+1}(t_\ell)$,

$\boldsymbol{x}_0(t_\ell) = [x_1(t_\ell), \cdots, x_M(t_\ell)]^T$.

Forward Recursion: For $i = 1, \cdots, M$:

$\boldsymbol{r}_{x_{i-1}d_{i-1}} = E[\boldsymbol{x}_{i-1}(t_\ell) d_{i-1}^*(t_\ell)]$;

$\delta_i = \|\boldsymbol{r}_{x_{i-1}d_{i-1}}\|_2, \ \ \boldsymbol{h}_i = \boldsymbol{r}_{x_{i-1}d_{i-1}}/\delta_i$;

$d_i(t_\ell) = \boldsymbol{h}_i^H \boldsymbol{x}_{i-1}(t_\ell), \ \sigma_{d_i}^2 = E[|d_i(t_\ell)|^2]$;

$\boldsymbol{x}_i(t_\ell) = \boldsymbol{x}_{i-1}(t_\ell) - \boldsymbol{h}_i d_i(t_\ell)$.

Backward Recursion: For $i = M, \cdots, 1$ with $\rho_M = E[|d_M(t_\ell)|^2]$ and $e_M(t_\ell) = d_M(t_\ell)$:

$w_i = \delta_i/\rho_i$;

$e_{i-1}(t_\ell) = d_{i-1}(t_\ell) - w_i^* e_i(t_\ell)$;

$\rho_{i-1} = \sigma_{d_{i-1}}^2 - |\delta_i|^2/\rho_i$.

Here we use $\|\cdot\|_2$ and $|\cdot|$ to denote the Euclidean norm of a vector and the absolute value of a number, respectively.

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