Angle Estimation via a Computationally Efficient SSF Method

Lei Huang, Shunjun Wu and Linrang Zhang
National Key Lab of Radar Signal Processing, Xidian University, Xi' an, Shaanxi 710071, P. R. China

Abstract: A computationally efficient method of signal subspace fitting (SSF) for angle estimation is proposed in this paper. Given the training data of one desired signal, the proposed method finds direction-of-arrival (DOA) parameters of all signals with much lower computational complexity than the classical weighted subspace fitting (WSF) method. Simulations are given to show that the proposed method provides the comparable estimation accuracy with the weighted subspace fitting estimator for uncorrelated and coherent signals.

Keywords: Multi-stage wiener filter, weighted subspace fitting, array signal processing, direction of arrival.

I. Introduction

Super-resolving correlated or even coherent signals is greatly interesting in the problem of direction-of-arrival (DOA) estimation. It is shown in [1] that the weighted subspace fitting (WSF) method [2]-[4], a minimizing technique, always outperforms the deterministic maximum likelihood (DML) estimator [5] in the performance of separating correlated signals. Nevertheless, the WSF estimator is of high computational complexity, partially due to the estimation of the signal subspace, which implies that the estimate of an array covariance matrix and its eigendecomposition are involved.

In the past two decades, one makes less use of the priori knowledge, more specifically, the training data or the spreading codes of desired signals, in DOA estimation. In fact, the priori knowledge is accessible in the modern communication system and GPS. Exploiting the priori knowledge, one may develop low computational complexity methods for DOA estimation with good estimation performance. It is well known that the multi-stage wiener filter (MSWF) proposed by Goldstein *et al* [6] [7] requires the training sequences or the spreading codes of users. As an efficient reduced-rank technique, the MSWF outperforms other adaptive reduced-rank methods such as the *Principal Components* (PC) method [8] and the *Cross-Spectral* (CS) metric [9]. The MSWF works so efficiently that it has been widely used in the interference suppression (IS) of communication [6] [10] and GPS [11] [12]. Recently, the methods termed the ROCK MUSIC [13][14] and ROCKET algorithms [15] based on the MSWF were proposed to high-resolution spectral estimation. However, the ROCK MUSIC technique requires the forward and backward recursions of the MSWF, which increase the complexity of the algorithm. Moreover, the ROCKET algorithm still needs complex matrix-matrix products to find the reduced-rank data matrix and the reduced-rank autoregressive (AR) weight vector. This indicates that additionally computational burden is included.

In the paper, we assume that the training data of one desired signal is well known. With the assumption, a computationally efficient method of signal subspace fitting (SSF) for DOA estimation is proposed. The proposed estimator does not compute the covariance matrix or its eigenvectors, and does not need the backward recursion of the MSWF, thereby requiring much lower computational cost than the existing subspace based methods. Numerical results imply that the estimation performance of the proposed estimator approaches to that of the classical WSF method for uncorrelated and coherent signals.

II. Problem formulation

A. Data Model

Let us consider a uniform linear array (ULA) of M isotropic sensors that received P narrowband signals from distinct directions $\theta_1, \theta_2, \cdots, \theta_P$. The data, which are corrupted by additive noise, received by the array at the kth snapshot can be written as

$$\begin{aligned} \mathbf{x}(k) &= \sum_{i=1}^{P} \mathbf{a}(\theta_i) s_i(k) + \mathbf{n}(k) \qquad (1) \\ &= \mathbf{A}(\theta)\mathbf{s}(k) + \mathbf{n}(k) \quad k = 0, \cdots, N-1 \end{aligned}$$

where

$$\begin{aligned} \mathbf{s}(k) &= [s_1(k), s_2(k), \cdots, s_P(k)]^T \\ \mathbf{A}(\theta) &= [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_P)] \end{aligned}$$

are the signal vector and the $M \times P$ steering matrix, respectively, $\mathbf{n}(k) \in \mathcal{C}^{M \times 1}$ is noise vector, N denotes the number of snapshots, P represents the number of signals, and the superscript $(\cdot)^T$ is the transpose operator. $\mathbf{a}(\theta_i)$ is the steering vector toward direction θ_i and takes the following form

$$\mathbf{a}(\theta_i) = \frac{1}{\sqrt{M}} \left[1, e^{j\varphi_i}, \cdots, e^{j(M-1)\varphi_i}\right]^T \qquad (2)$$

where $\varphi_i = \frac{2\pi d}{\lambda} \sin(\theta_i)$ in which $\theta_i \in (-\pi/2, \pi/2)$, d and λ are inter-element spacing and the wavelength, respectively. Throughout the paper we assume that $M > P$. Furthermore, the background noise uncorrelated with the signals is a stationary Gaussian white random process, which is also spatially white and circularly symmetric. Therefore, the covariance matrix can be expressed as

$$\mathbf{R_x} = E\big[\mathbf{x}(k)\mathbf{x}^H(k)\big] = \mathbf{A}(\theta)\mathbf{R_s}\mathbf{A}^H(\theta) + \sigma_{\mathbf{n}}^2\mathbf{I}_M \qquad (3)$$

where $\mathbf{R_s} = E\left[\mathbf{s}(k)\mathbf{s}^H(k)\right]$ and $\sigma_{\mathbf{n}}^2$ are the signal covariance matrix and the noise variance, respectively, and $\mathbf{I}_M$ is the $M \times M$ identity matrix.

For uncorrelated signals, performing the eigenvalue decomposition of the covariance matrix $\mathbf{R_x}$ leads to

$$\mathbf{R_x} = \mathbf{V_s}\mathbf{\Lambda_s}\mathbf{V_s}^H + \sigma_{\mathbf{n}}^2\mathbf{V_n}\mathbf{V_n}^H. \qquad (4)$$

The number of the columns of $\mathbf{V_s}$ is equal to the rank P' of the signal covariance matrix $\mathbf{R_s}$. Thus the columns of $\mathbf{V_s}$ span the same range subspace of $\mathbf{A}(\theta)$. Considering (3) and (4), and performing some algebraic manipulations yield

$$\mathbf{V_s} = \mathbf{A}(\theta)\mathbf{Q} \qquad (5)$$

where $\mathbf{Q} \in C^{P \times P'}$ is the nonsingular matrix. Equation (5) forms a basis for the classical signal subspace fitting. θ and $\mathbf{Q}$ are unknown and require to be searched by solving (5). In fact, if the theoretical $\mathbf{V_s}$ is replaced by its estimate $\hat{\mathbf{V}}_\mathbf{s}$, there will be no accurate solution to the equation above. In this case, one attempts to minimize some distance measure between $\hat{\mathbf{V}}_\mathbf{s}$ and $\mathbf{A}(\theta)\mathbf{Q}$. For this purpose, the Frobenius norm is often used. Therefore, the SSF estimator is obtained by solving the following non-linear optimization problem:

$$\{\hat{\theta}, \hat{\mathbf{Q}}\} = \arg\min_{\theta, \mathbf{Q}} \|\hat{\mathbf{V}}_\mathbf{s} - \mathbf{A}(\theta)\mathbf{Q}\|_F^2. \qquad (6)$$

Since the cost function above is quadratic with respect to $\mathbf{Q}$, $\hat{\mathbf{Q}}$ is easily obtained. By substituting the least squares solution $\hat{\mathbf{Q}} = [\mathbf{A}^H(\theta)\mathbf{A}(\theta)]^{-1}\mathbf{A}^H(\theta)\hat{\mathbf{V}}_\mathbf{s}$ into (6), we obtain the following equivalent optimization problem without the parameter $\mathbf{Q}$:

$$\hat{\theta}_{SSF} = \arg\min_{\theta}\left\{\mathrm{tr}\left[\mathbf{P}_\mathbf{A}^\perp(\theta)\hat{\mathbf{V}}_\mathbf{s}\hat{\mathbf{V}}_\mathbf{s}^H\right]\right\} \qquad (7)$$

where $\mathbf{P}_\mathbf{A}^\perp = \mathbf{I}_M - \mathbf{A}(\theta)[\mathbf{A}^H(\theta)\mathbf{A}(\theta)]^{-1}\mathbf{A}^H(\theta)$. Since the eigenvectors are estimated with a quality, commensurate with the closeness of the corresponding eigenvalues to the noise variance, it is natural to weight each eigenvectors and lead to

$$\hat{\theta}_{WSF} = \arg\min_{\theta}\left\{\mathrm{tr}\left[\mathbf{P}_\mathbf{A}^\perp(\theta)\hat{\mathbf{V}}_\mathbf{s}\mathbf{W}\hat{\mathbf{V}}_\mathbf{s}^H\right]\right\} \qquad (8)$$

where $\mathbf{W}$ is the weighting matrix whose *optimal* solution is $\mathbf{W}_{opt} = (\mathbf{\Lambda_s} - \sigma_{\mathbf{n}}^2\mathbf{I}_M)^2\mathbf{\Lambda_s}^{-1}$ [1].

B. Multi-Stage Wiener Filter

It is well known that the wiener filter (WF) $\mathbf{w}_{wf} \in \mathcal{C}^{M\times 1}$ can be used to estimate the desired signal $d(k) \in \mathcal{C}$ from the observation data $\mathbf{x}(k)$ in the minimum mean square error (MMSE) sense. Thereby, we get the following design criterion

$$\mathbf{w}_{wf} = \arg\min_{\mathbf{w}} E\{|d(k) - \mathbf{w}^H\mathbf{x}(k)|^2\} \qquad (9)$$

where $\hat{d}(k) = \mathbf{w}^H\mathbf{x}(k)$ represents the estimate of the desired signal $d(k)$, and $\mathbf{w} \in \mathcal{C}^{M\times 1}$ is the linear filter. Solving (9) leads to the Wiener-Hopf equation

$$\mathbf{R_x}\mathbf{w}_{wf} = \mathbf{r_{x}}_d \qquad (10)$$

where $\mathbf{r_x}_d = E[\mathbf{x}(k)d^*(k)]$. The classical wiener filter, *i.e.*, $\mathbf{w}_{wf} = \mathbf{R_x}^{-1}\mathbf{r_x}_d$, is computationally intensive for large M since the inverse of the covariance matrix is required. The MSWF was developed by Goldstein *et al* [7] to find an approximate solution to the Wiener-Hopf equation which does not need the inverse of the covariance matrix. In contrast to the *principal components* (PC) method [8] and the *cross-spectral* (CS) metric [9], the MSWF needs much lower computational cost, offers faster convergence and can work in the low-sample support operational environment where other adaptive algorithms fail. The MSWF based on the data-level lattice structure [16] is given as follows:

- *Initialization*: $d_0(k) = s_1(k)$ and $\mathbf{x}_0(k) = \mathbf{x}(k)$.
- *Forward Recursion*: For $i = 1, 2, \cdots, M$:
  $\mathbf{h}_i = E[\mathbf{x}_{i-1}(k)d_{i-1}^*(k)]/\|E[\mathbf{x}_{i-1}(k)d_{i-1}^*(k)]\|_2$;
  $d_i(k) = \mathbf{h}_i^H\mathbf{x}_{i-1}(k)$;
  $\mathbf{x}_i(k) = \mathbf{x}_{i-1}(k) - \mathbf{h}_i d_i(k)$.
- *Backward Recursion*: For $i = M, M-1, \cdots, 1$ with $e_M(k) = d_M(k)$:
  $w_i = E[d_{i-1}(k)e_i^*(k)]/E[|e_i(k)|^2]$;
  $e_{i-1}(k) = d_{i-1}(k) - w_i^* e_i(k)$.

The corresponding block diagram is seen in Fig. 1.

The pre-filtering matrix $\mathbf{T}_M = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_M]$ is formed

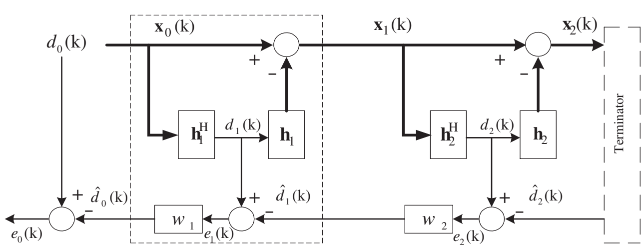

Fig. 1. Lattice structure of the MSWF. The dashed line denotes the basic box for each additional stage.

by the M matched filters in the forward decomposition of the MSWF. Notice that the matched filter $\mathbf{h}_i \in \mathcal{C}^M, i = 1, 2, \ldots, M$ maximizes the real part of the correlation between the new desired signal $d_i(k) = \mathbf{h}_i^H x_{i-1}(k) \in \mathcal{C}$ at stage i and the desired signal $d_{i-1}(k)$ at the previous stage $i-1$, forcing the desired signals between successive stages to be in-phase. However, the blocking matrix $\mathbf{B}_i = \mathbf{I}_M - \mathbf{h}_i\mathbf{h}_i^H$ guarantees that $\mathbf{T}_M$ decorrelates all lags in the process $d_i(k)$ greater than one. It follows that the pre-filtered covariance matrix is tridiagonal, $i.e.$

$$\mathbf{T}_M^H\mathbf{R}_{\mathbf{x}_0}\mathbf{T}_M = \begin{bmatrix} \sigma_{d_1}^2 & \delta_2^* & & & \\ \delta_2 & \sigma_{d_2}^2 & \delta_3^* & & \\ & \delta_3 & \sigma_{d_3}^2 & \ddots & \\ & & \ddots & \ddots & \delta_M^* \\ & & & \delta_M & \sigma_{d_M}^2 \end{bmatrix} := \mathbf{R}_d \quad (11)$$

where where $\sigma_{d_i}^2 = E\left[d_i(k)d_i^*(k)\right]$, and $\delta_i = E\left[d_i(k)d_{i-1}^*(k)\right]$.

III. Computationally Efficient SSF Method

Notice that the pre-filtering matrix $\mathbf{T}_M$ is the unitary matrix. Thereby, considering (3) and (11) results in

$$\begin{aligned} \mathbf{A}\mathbf{R}_\mathbf{s}\mathbf{A}^H + \sigma_\mathbf{n}^2\mathbf{I}_M &= \mathbf{T}_M\mathbf{R}_d\mathbf{T}_M^H \\ &= \left[\begin{array}{cc}\mathbf{T}_\mathbf{s} & \mathbf{T}_\mathbf{n}\end{array}\right]\begin{bmatrix}\mathbf{R}_d^\mathbf{s} & \mathbf{C}^H \\ \mathbf{C} & \mathbf{R}_d^\mathbf{n}\end{bmatrix}\begin{bmatrix}\mathbf{T}_\mathbf{s}^H \\ \mathbf{T}_\mathbf{n}^H\end{bmatrix} \\ &= \mathbf{T}_\mathbf{s}\mathbf{R}_d^\mathbf{s}\mathbf{T}_\mathbf{s}^H + \mathbf{T}_\mathbf{n}\mathbf{C}\mathbf{T}_\mathbf{s}^H \\ &+ \mathbf{T}_\mathbf{s}\mathbf{C}^H\mathbf{T}_\mathbf{n}^H + \mathbf{T}_\mathbf{n}\mathbf{R}_d^\mathbf{n}\mathbf{T}_\mathbf{n}^H \\ &= \mathbf{T}_\mathbf{s}\mathbf{R}_d^\mathbf{s}\mathbf{T}_\mathbf{s}^H + \mathbf{T}_\mathbf{n}\mathbf{R}_d^\mathbf{n}\mathbf{T}_\mathbf{n}^H \\ &+ \delta_{P'+1}\mathbf{h}_{P'+1}\mathbf{h}_{P'}^H + \delta_{P'+1}^*\mathbf{h}_{P'}\mathbf{h}_{P'+1}^H \end{aligned} \quad (12)$$

where

$$\mathbf{C} = \begin{bmatrix} 0 & \cdots & 0 & \delta_{P'+1} \\ 0 & \cdots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \end{bmatrix} \quad (13)$$

$$\mathbf{T}_\mathbf{s} = [\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_{P'}] \quad (14)$$

$$\mathbf{T}_\mathbf{n} = [\mathbf{h}_{P'+1}, \mathbf{h}_{P'+2}, \cdots, \mathbf{h}_M]. \quad (15)$$

It should be mentioned that $\mathbf{R}_d$ can be expressed as

$$\mathbf{R}_d = E\left[\mathbf{d}(k)\mathbf{d}^H(k)\right] \quad (16)$$

where

$$\mathbf{d}(k) = [d_1(k), d_2(k), \cdots, d_M(k)]^T. \quad (17)$$

Since the process $\mathbf{x}_i(k), i \in \{P', P'+1, \cdots, M-1\}$ is white and has the following form

$$\mathbf{x}_i(k) = \left(\prod_{j=i}^{1}\mathbf{B}_j\right)\mathbf{n}(k) \quad (18)$$

where $\mathbf{B}_j = \mathbf{I}_M - \mathbf{h}_j\mathbf{h}_j^H$. It follows that

$$\begin{aligned} \delta_{i+1} &= E\left[d_{i+1}(k)d_i^H(k)\right] \\ &= E\left[\mathbf{h}_{i+1}^H\mathbf{x}_i(k)\mathbf{x}_{i-1}^H(k)\mathbf{h}_i\right] \\ &= \mathbf{h}_{i+1}^H E\left[\mathbf{x}_i(k)\mathbf{x}_{i-1}^H(k)\right]\mathbf{h}_i \\ &= 0 \end{aligned} \quad (19)$$

where $i = P', P'+1, \cdots, M-1$. So (12) can be rewritten as

$$\mathbf{A}(\theta)\mathbf{R}_\mathbf{s}\mathbf{A}^H(\theta) + \sigma_\mathbf{n}^2\mathbf{I}_M = \mathbf{T}_\mathbf{s}\mathbf{R}_d^\mathbf{s}\mathbf{T}_\mathbf{s}^H + \mathbf{T}_\mathbf{n}\mathbf{\Lambda}_\mathbf{n}\mathbf{T}_\mathbf{n}^H \quad (20)$$

where $\mathbf{\Lambda}_\mathbf{n} = diag(\sigma_{d_{P'+1}}^2, \sigma_{d_{p'+2}}^2, \cdots, \sigma_{d_M}^2)$. It is easy to see that $\mathbf{T}_\mathbf{n}^H\mathbf{T}_\mathbf{s} = 0$ since all the matched filters $\mathbf{h}_1, \mathbf{h}_2, \cdots, \mathbf{h}_M$ are orthogonal. Post-multiplying two sides of (20) by $\mathbf{T}_\mathbf{s}$ leads to

$$\begin{aligned} \mathbf{A}(\theta)\mathbf{R}_\mathbf{s}\mathbf{A}^H(\theta)\mathbf{T}_\mathbf{s} + \sigma_\mathbf{n}^2\mathbf{T}_\mathbf{s} &= \mathbf{T}_\mathbf{s}\mathbf{R}_d^\mathbf{s}\mathbf{T}_\mathbf{s}^H\mathbf{T}_\mathbf{s} + \mathbf{T}_\mathbf{n}\mathbf{\Lambda}_\mathbf{n}\mathbf{T}_\mathbf{n}^H\mathbf{T}_\mathbf{s} \\ &= \mathbf{T}_\mathbf{s}\mathbf{R}_d^\mathbf{s}, \end{aligned} \quad (21)$$

namely

$$\mathbf{T}_\mathbf{s}(\mathbf{R}_d^\mathbf{s} - \sigma_\mathbf{n}^2\mathbf{I}_{P'}) = \mathbf{A}(\theta)\mathbf{R}_\mathbf{s}\mathbf{A}^H(\theta)\mathbf{T}_\mathbf{s}. \quad (22)$$

Therefore

$$\begin{aligned} \mathbf{T}_\mathbf{s} &= \mathbf{A}(\theta)\mathbf{R}_\mathbf{s}\mathbf{A}^H(\theta)\mathbf{T}_\mathbf{s}(\mathbf{R}_d^\mathbf{s} - \sigma_\mathbf{n}^2\mathbf{I}_{P'})^{-1} \\ &= \mathbf{A}(\theta)\mathbf{K} \end{aligned} \quad (23)$$

where $\mathbf{K} = \mathbf{R}_\mathbf{s}\mathbf{A}^H(\theta)\mathbf{T}_\mathbf{s}(\mathbf{R}_d^\mathbf{s} - \sigma_\mathbf{n}^2\mathbf{I}_{P'})^{-1} \in \mathcal{C}^{P\times P'}$ Since $\mathbf{R}_\mathbf{s}$ and $(\mathbf{R}_d^\mathbf{s} - \sigma_\mathbf{n}^2\mathbf{I}_{P'})^{-1}$ are the nonsingular matrices and $\mathbf{A}(\theta)$ is the vandermonde matrix, we get

$$rank(\mathbf{K}) = rank\left[\mathbf{A}^H(\theta)\mathbf{T}_\mathbf{s}\right] = P'. \quad (24)$$

Therefore, from (23) it follows that $\mathbf{T}_\mathbf{s}$ spans the signal subspace. Thereby, the relation (23) creates a basis for the SSF, and we have the following criterion function:

$$\left\{\hat{\theta}, \hat{\mathbf{K}}\right\} = \arg\min_{\theta,\mathbf{K}} \|\hat{\mathbf{T}}_\mathbf{s} - \mathbf{A}(\theta)\mathbf{K}\|_F^2 \quad (25)$$

where $\hat{\mathbf{T}}_\mathbf{s}$ is the estimate of $\mathbf{T}_\mathbf{s}$.

Similar to (6), (25) is also quadratic with respect to $\mathbf{K}$. Thus, the parameter $\mathbf{K}$ can be solved and replaced in the criterion function above. For the fixed unknown parameter $\mathbf{A}(\theta)$, the solution for the linear parameter $\mathbf{K}$ is

$$\hat{\mathbf{K}} = \mathbf{A}^\dagger(\theta)\hat{\mathbf{T}}_\mathbf{s} \quad (26)$$

where $\mathbf{A}^\dagger(\theta) = \left[\mathbf{A}^H(\theta)\mathbf{A}(\theta)\right]^{-1}\mathbf{A}^H(\theta)$. Substituting (26) into (25), we get the SSF cost function without $\mathbf{K}$:

$$\begin{aligned} \hat{\theta} &= \arg\min_\theta \|\mathbf{P}_\mathbf{A}^\perp(\theta)\hat{\mathbf{T}}_\mathbf{s}\|_F^2 \\ &= \arg\min_\theta \left\{tr\left[\mathbf{P}_\mathbf{A}^\perp(\theta)\hat{\mathbf{T}}_\mathbf{s}\hat{\mathbf{T}}_\mathbf{s}^H\right]\right\}. \end{aligned} \quad (27)$$

From (27), it is easy to see that, the new signal subspace is spanned by the former P' matched filters of the MSWF. Finding the novel signal subspace only needs the P' forward recursions of the MSWF, dose not require to estimate the array covariance matrix $\mathbf{R}_{\mathbf{x}_0}$ and compute its eigenvectors. Hence, the novel criterion function is very distinct from that of the classical WSF estimator though they are similar formally.

IV. Computational Cost Requirement

It should be noted that the efficient implementation of the MSWF based on the data-lever lattice structure avoids the formation of blocking matrices, and all the operations of the MSWF only involve complex matrix-vector products, thereby requiring the computational complexity of $O(MN)$ for each matched filter $\mathbf{h}_i, i = 1, 2, \cdots, P'$. Thus, to estimate the signal subspace $\mathbf{T}_\mathbf{s}$ of rank P', the computational cost of the proposed method is merely $O(P'MN)$ flops. However, the classical WSF estimator relays on the estimation of the covariance matrix and its eigendecomposition, which need $O(M^2N + M^3)$ flops.

V. NUMERICAL RESULTS

In this section, the performance of the proposed method and the classical WSF method is compared for the problem of the DOA estimation. The array herein is assumed to be a ULA with the isotropic sensors, whose spacings equal half-wavelength. The number of signals is known. We make 300 Monte Carlo runs for each experiment to compute the root-mean-squared errors (RMSE's) of estimated DOAs.

Example 1 (uncorrelated signal case) Suppose that there are three uncorrelated signals with equal power in the far field impinging upon the ULA. The true DOAs are $\{-4^{\mathrm{o}}, 0^{\mathrm{o}}, 5^{\mathrm{o}}\}$. The background noise is assumed to be a stationary Gaussian white random process. Signal-to-noise ratio (SNR) is defined as $10\log(\sigma_{\mathbf{s}}^2/\sigma_{\mathbf{n}}^2)$, where $\sigma_{\mathbf{s}}^2$ is the power of each signal in single sensor. The RMSE's of estimated DOAs versus SNR are shown in Fig. 2, where the rank of the MSWF is equal to 3, the number of sensors is 16 and the number of snapshots equals 64. It is demonstrated in Fig. 2 that the new estimator yields the comparable estimation performance as the WSF method when SNR is greater than 10dB, the performance of the proposed method slightly decreased as SNR$\leq$ 10dB. The RMSE's of the two estimators approach to the Cramér-Rao bound (CRB) as SNR is large.

The RMSE's of estimated DOAs versus the number of snapshots are shown in Fig. 3, where SNR=15dB and D=3. It can be observed that the RMSE's of the proposed method almost coincide with those of the classical WSF as the number of snapshots increases, thereby indicating that the proposed method yields the comparable estimation accuracy with the WSF method.

Example 2 (coherent signal case) Suppose that there are three signals impinging upon the array from the same signal source. The first is a direct-path signal and the others are the scaled and delayed replicas of the first signal that represent the multipaths or the "smart" jammers. The propagation constants are $\{1, -0.8 + j0.6, -0.4 + j0.7\}$. The true DOAs are still assumed to be $\{-4^{\mathrm{o}}, 0^{\mathrm{o}}, 5^{\mathrm{o}}\}$. The RMSE's of estimated DOAs versus SNR are demonstrated in Fig. 4 where the rank of the MSWF equals 1 and the number of snapshots is 64. It is easy to see that the proposed method outperforms the WSF estimator when SNR is less than -5dB. When SNR is large, the RMSE's of the two methods approach to the CRB.

For the fixed SNR equal to 15dB and the rank of the MSWF equal to 1, Fig. 5 displays that the RMSE's of the proposed method are lower than or coincide with those of the classical WSF method as the number of snapshots increases, thus implying that the proposed technique surpasses the classical WSF method in the case of coherent signals.

VI. CONCLUSION

A computationally efficient SSF method for DOA estimation has been presented in the paper. The new signal subspace of the proposed method is obtained merely by calculating the former P' matched filters of the MSWF, dose not include the estimate of the covariance matrix or its eigendecomposition, either not need to compute the scalar wiener filters in the backward recursion of the MSWF. Thus, the computational complexity of the proposed method is much lower than that of the classical WSF method. Numerical results shows that the proposed estimator yields the comparable estimation accuracy with the classical WSF method.

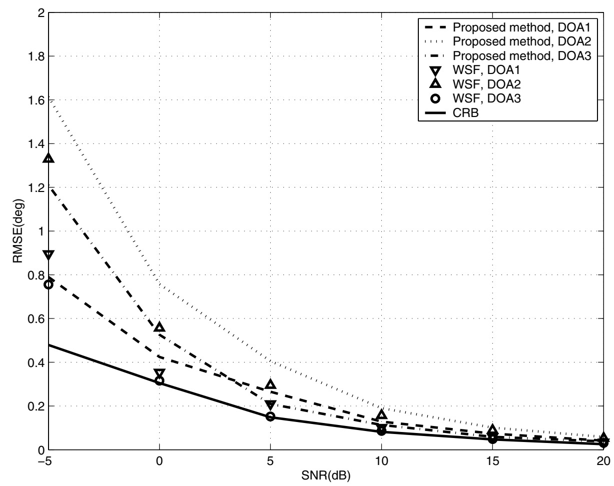

Fig. 2. RMSE's of estimated DOAs versus SNR for fixed N=64, M=16 and D=3.

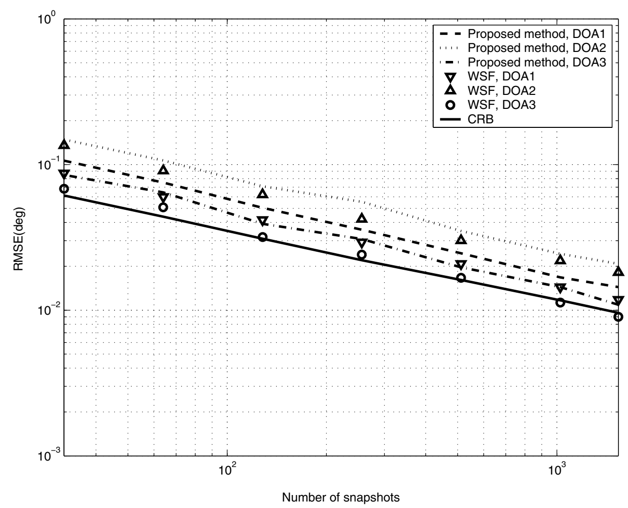

Fig. 3. RMSE's of estimated DOAs versus number of snapshots for fixed SNR=15dB, M=16 and D=3.

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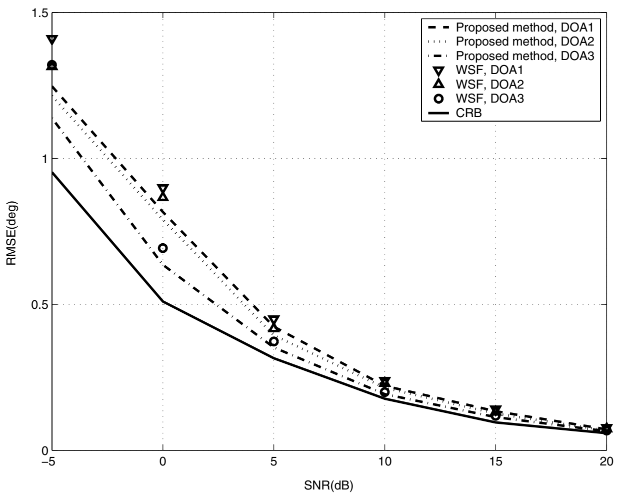

Fig. 4. RMSE's of estimated DOAs versus SNR for fixed N=64, M=16 and D=1.

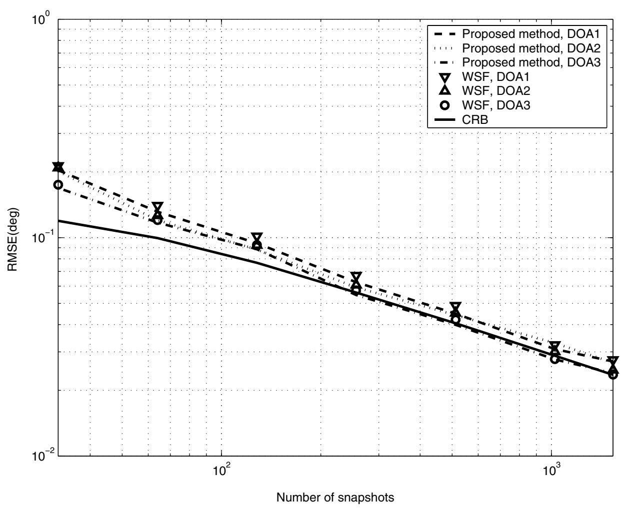

Fig. 5. RMSE's of estimated DOAs versus number of snapshots for fixed SNR=15dB, M=16 and D=1.

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